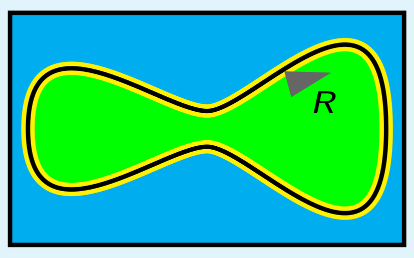
Reinforced separators for a new guaranteed set projection algorithm

Nuwan Herath Mudiyanselage, joint work with Luc Jaulin and Simon Rohou

ENSTA Bretagne, Lab-STICC, UMR CNRS 6285, Brest, France

Context

Solving engineering problems in robotics or in control theory requires guaranteed solution sets. Those results must handle uncertainties due to imprecise measurementss and numerical limitations.
One could want the **guaranteed projection of a set**. For example, one may only be interested by the position (x, y) while the problem is defined by x, y, $p_1, \dots, p_n$, where the p_i are some parameters.



Is the robot R inside the safe region (in the green region)?

Interval arithmetic

$[x] = [x^-, x^+]$, $[y] = [y^-, y^+]$

- $[x] + [y] = [x^- + y^-, x^+ + y^+]$
- $[x] - [y] = [x^- - y^+, x^+ - y^-]$
- ...

⚠overestimation due to interval dependency

$\underbrace{[-1,1]\cdot[-1,1]}_{[-1,1]} \supset \underbrace{[-1,1]^2}_{[0,1]}$

Extension to vectors

$$[v] = \begin{pmatrix}[v_1^-, v_1^+] \\ [v_2^-, v_2^+]\end{pmatrix}$$

Extension to sets

$[\mathbb{X}] = [\mathbb{X}^-, \mathbb{X}^+]$

Set descriptor

Goal: approximate a set $\mathbb{X}$ with two unions of boxes $\mathbb{X}^-$ and $\mathbb{X}^+$, such that $\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+$

$\mathbb{X}$ set (in $\mathcal{P}(\mathbb{R}^n)$)

$[x]$ box (in $\mathbb{IR}^n$)

$\mathcal{C}$ contractor

$\mathcal{S}$ separator

NEW

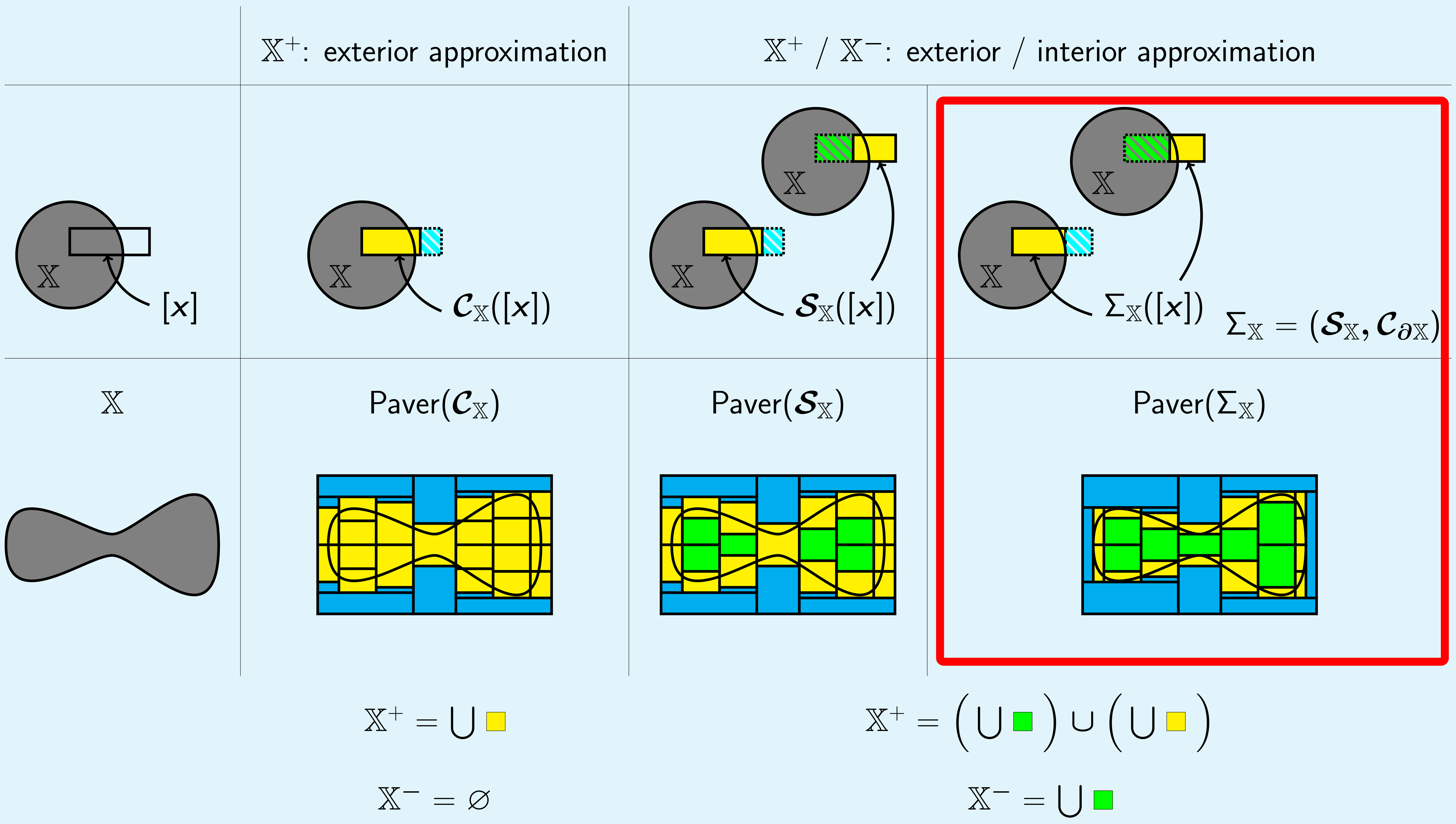
Σ boundary reinforced separator

$\bigcup$ ■ (green) $\subset \mathbb{X}$

$\bigcup$ ■ (blue) $\subset \overline{\mathbb{X}}$

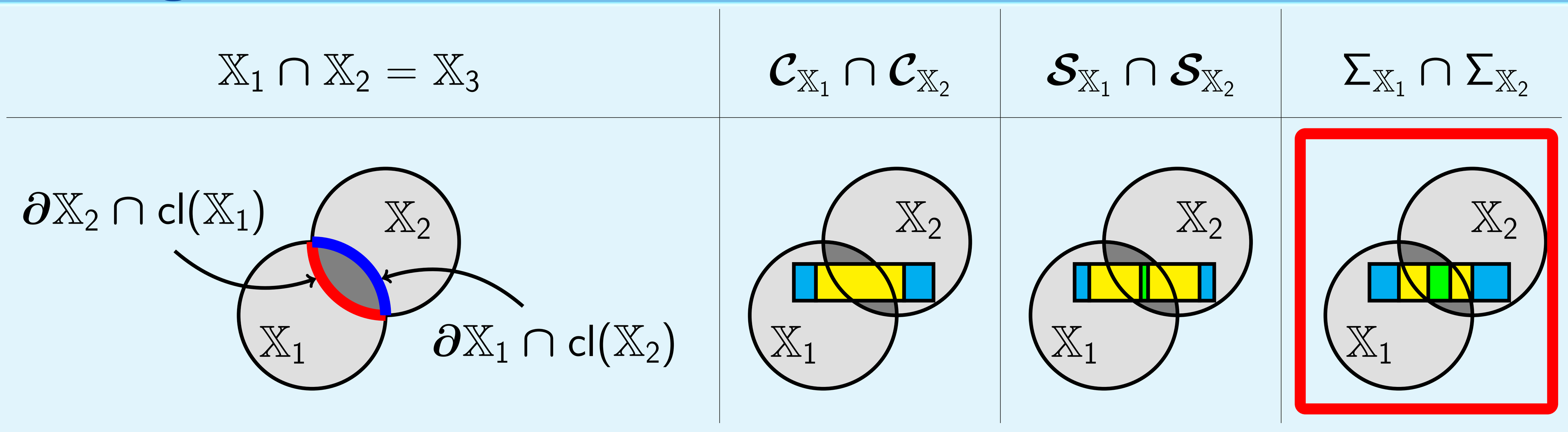
■ (yellow) undecided box

$\left(\bigcup \text{■ (yellow)} \supset \partial\mathbb{X}\right)$



[Gilles Chabert and Luc Jaulin, Contractor programming, Artificial Intelligence, Volume 173, Issue 11, 2009]

Contractor and separator algebra



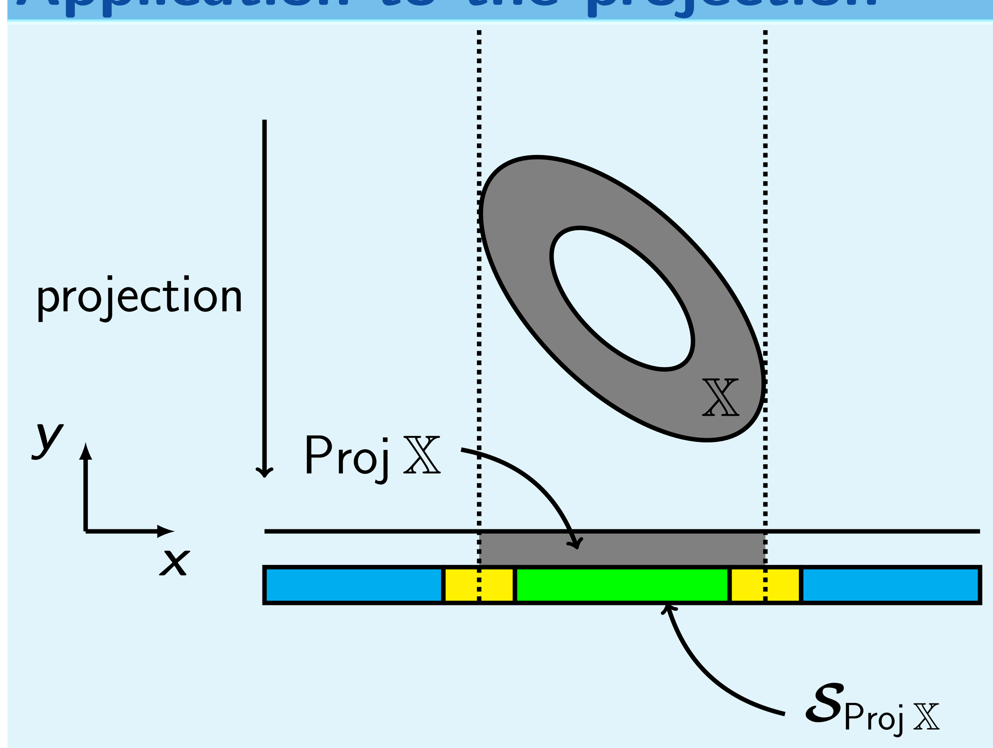
$$\Sigma_{\mathbb{X}_1} \cap \Sigma_{\mathbb{X}_2} = \Sigma_{\mathbb{X}_3} = (\mathcal{S}_{\mathbb{X}_3}, \mathcal{C}_{\mathbb{X}_3})$$

with

$$\mathcal{S}_{\mathbb{X}_3} = \mathcal{S}_{\mathbb{X}_1} \cap \mathcal{S}_{\mathbb{X}_2}$$
$$\mathcal{C}_{\mathbb{X}_3} = \phi(\mathcal{S}_{\mathbb{X}_1}, \mathcal{S}_{\mathbb{X}_2}, \mathcal{C}_{\partial\mathbb{X}_1}, \mathcal{C}_{\partial\mathbb{X}_2})$$

[Luc Jaulin and Benoît Desrochers, Introduction to the algebra of separators with application to path planning, Engineering Applications of Artificial Intelligence, Volume 33, 2014]

Application to the projection



$\mathcal{S}_{\text{Proj}\,\mathbb{X}} = \text{SepProj}(\mathcal{S}_\mathbb{X})$

Projection of the following set

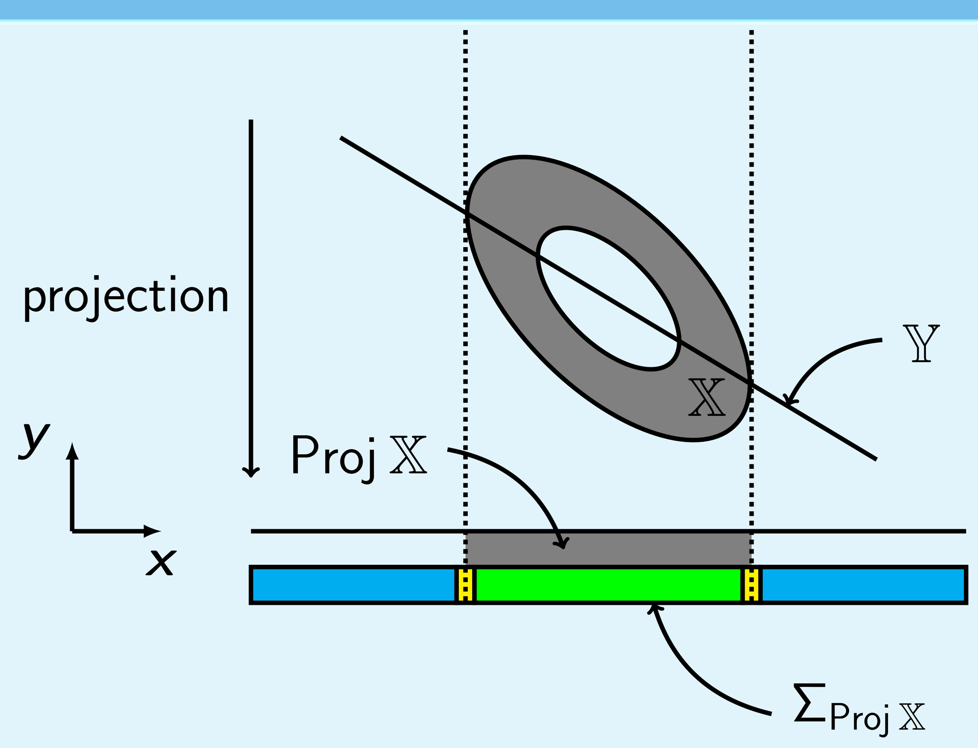
$$\mathbb{X} = \left\{ (x, y) \in \mathbb{R}^2 \;\middle|\; f(x, y) \in [a, b] \right\}$$
$$f(x, y) = x^2 + xy + y^2$$

It is described by $\mathcal{S}_\mathbb{X}$

The locii of the vertical tangents are in

$$\mathbb{Y} = \left\{ (x, y) \in \mathbb{R}^2 \;\middle|\; \frac{\partial f}{\partial y}(x, y) = 0 \right\}$$

It is described by $\mathcal{C}_\mathbb{Y}$, which helps to reinforce the separator of the projection



$\Sigma_{\text{Proj}\,\mathbb{X}} = \text{SepProj}(\Sigma_\mathbb{X}, \mathcal{C}_\mathbb{Y})$

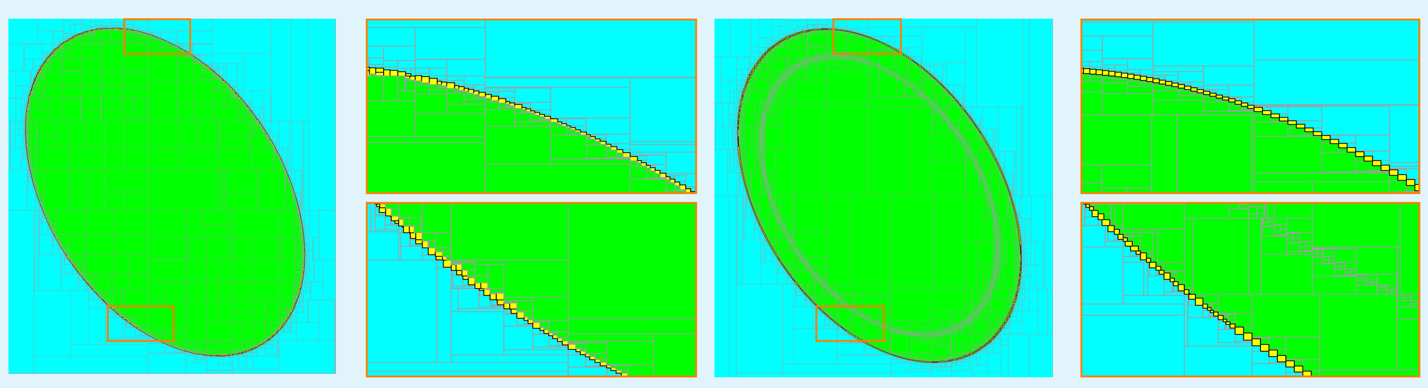
A toy set projection problem

```
from codac import *

# Input
interval = Interval(-0.3, 0)
f = Function("x", "y", "z", "2*x^2 + x*y + x*z + y^2 + z^2 - 1")
df = Function("x", "y", "z", "x + 2*z")

# Projection separators
sep = SepProj(SepFunction(f, interval))
br_sep = SepProj(BRSepFunction(f, interval), CtcFunction(df))

# Paving
Paver(sep)
Paver(br_sep)
```



(a) (b) (c) (d)

(a) and (b): $\mathcal{S}_{\text{Proj}\,\mathbb{X}}$ (SepFunction)
(c) and (d): $\Sigma_{\text{Proj}\,\mathbb{X}}$ (BRSepFunction + CtcFunction)

[Simon Rohou and Benoît Desrochers and others, The Codac library – Constraint-programming for robotics, http://codac.io, 2022]